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Plato, Diagrammatic Reasoning and Mental Models

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PLATO, DIAGRAMMATIC REASONING AND MENTAL MODELS

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Republic, VI 509d-511

Objects: Ideas or Forms Lower Noetics

Rational Understanding faculties: noēsis ນວ່າງວາງ

A

Thought *dianoia* διάνοια

B

Intelligible World

Physical Objects Images

С

(shadows, reflections)

D

BeliefImaginationpistiseikasiaπίστιςεἰκασία

Visible World

Representation systems:

linguistic

diagrammatic

THE DIVIDED LINE: THE STANDARD REPRESENTATION

THE DIVIDED LINE: THE STANDARD INTERPRETATIONS

• The Demarcation Interpretation:

«The middle two subsegments were not meant to be compared» (Foley 2008, 9).

Its name derives from the fact that its exponents think that exists a «clear *demarcation* between the intended and unintended points of comparison, and such a demarcation will show that the **equality of the middle subsegments** can be dismissed because it falls into the latter category» (Foley 2008, 10).

• The Gaffe Interpretation:

«The **equality of the middle subsegments** is a minor unintended implication of the division procedure...Plato was simply unaware that he had given contradictory instructions for the division » (Foley 2008, 12-13).

THE DIVIDED LINE: THE COLLABORATION BETWEEN WRITER AND READER

• Plato, after having summarized his idea of cognitive progress, schematized using the divided line, tells his readers that there is much more to know about the subject than what had been discussed so far with Glaucon (*Republic*, VII 534 a):

But as for the ratios between the things these are set over and the division of either the opinable or the intelligible section into two, let's pass them by, Glaucon, *lest they involve us in arguments many times longer than the ones we have already gone through* (My emphasis)

• Foley (2008, 23), commenting on the previous excerpt from the *Republic*, emphasizes:

the passage shows that Plato is not willing to set forth his views on the further complexities that have emerged. It is a task that he *intentionally* leaves for his readers, revealing that his final assessment of the role of the divided line is to *force a thoughtful reader to transcend the text*. One significant aspect of the divided line is exactly that Plato refuses to explain its point. (Foley 2008, 23. My emphasis)

THE DIVIDED LINE: THE COLLABORATION BETWEEN WRITER AND READER

- As the whole nature is akin, and the soul has learned everything, nothing prevents a man, after recalling one thing only-a process men call **learning**-discovering everything else for himself, if he is brave and does not tire of the search, for searching and learning are, as a whole, **recollection**. (*Meno*, 81 c-d. My emphasis)
- O most expert Theuth, one man can give birth to the elements of an art, but only another can judge how they can benefit or harm those who will use them. And now, since you are the father of **writing**, your affection for it has made you describe its effects as the opposite of what they really are. In fact, it will introduce forgetfulness into the soul of those who learn it: they will not practice using their memory because they will put their trust in writing, which is **external** and depends on **signs that belong to others**, instead of trying to remember from the inside, completely on their own. You have **not** discovered a potion for **remembering**, but for **reminding**; you provide your students with the **appearance of wisdom**, not with its reality. Your invention will enable them to hear many things without being properly taught, and they will imagine that they have come to know much while for the most part they know nothing. And they will be difficult to get along with, since they will merely appear to be wise instead of really being so. (*Phaedrus*, 275 a-b. My emphasis)

THE DIVIDED LINE: THE COLLABORATION BETWEEN WRITER AND READER

- Plato's written words are **reminders**
- Plato's written words are *not* the *final* destination of knowledge, but rather a stimulation to reach that destination

Plato calls for a collaboration between writer and *reader*: Plato has *not* written a *textbook* whose content can be merely summarized by the readers. He has created a *text to which they are required to respond and the act of responding to the text is important as the text itself.*

- A B C D A' B' C' D'
- I do think that the *basics* of Plato's thought *are* in the written dialogues but the dialogues should *not* be considered as the *final* stage of cognitive evolution

Republic, VII 534 a: *larger cognitive project* indicated by Plato

• *My* reconstruction of the stages of human intellectual development

scientific method: In science, when there are testable elements that present variations which are not in line with what was theorized about their properties, it is possible, before rejecting the theories about those elements, to hypothesize that the unpredictable variations are generated by other elements, whose existence was not taken into consideration before.

MY COLLABORATION WITH THE TEXT:

MY REPRESENTATION OF THE DIVIDED LINE

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scientific method: in our case, *Republic*, VII 534 a is the unpredictable effect which confirms my theory that the *basics* of Plato's thought *are* in the written dialogues but the dialogues should *not* be considered as the *final* stage of cognitive development; rather, there are stages of cognitive development, which add subsections to the line segment used by Plato to represent intellectual progress. These subsections are indicated with the letters A', B', C', D' in the schematization above and they are those elements, which have not been taken into consideration before.

 In my representation, all the sectors of the line segment have equal lenght:

This does *not* mean that I think that there is **no theoretical difference** among the objects and rational faculties that can be located within the line segment. **MY COLLABORATION WITH THE TEXT:**

In my representation, all the sectors of the line segment have equal length because I want to emphasize the equal epistemic significance that *each* subsection has for the individual rational development.

- In my line segment, the subsection A represents the images and the cognitive capacity necessary to grasp them. When we are able to understand D, the Ideas, we reach a superior level of intellectual development. Starting from this epistemic moment, we are able to begin the investigation of the purely intelligible, which is for Plato the highest rational achievement.
- I have hypothesized that there can be stages of rational progress also in the cognitive development of the individuals who are already able to investigate the purely intelligible.

MY COLLABORATION WITH THE TEXT:

The four sectors of the line segment traced in *Republic*, VI 509d-511: (A-D): *Theoretical Childhood*

The larger cognitive project indicated by Plato in *Republic*, VII 534a: (A'- D'): *Theoretical Adulthood*

With the expressions theoretical childhood and theoretical children, I am *not* referring to *real* children and their cognitive development but I am defining **phases of rational evolution**, **one intellectually more advanced than the other**, coherent with Plato's indications.

MY COLLABORATION WITH THE TEXT:

The four sectors of the line segment traced in *Republic*, VI
 509d-511(A-D): *Theoretical Childhood*: *natural language Phaedrus*, 275 a-b written reminders

The larger cognitive project indicated by Plato in *Republic*, VII 534a: (A'- D'): *Theoretical Adulthood*: *mathematics*

• TWO LEVELS OF MATHEMATICAL COMPLEXITY:

Top-down axiomatic approach standard
 representation of the divided line: lower noetics ;
 lower noetics = mathematical objects

MY COLLABORATION WITH THE TEXT:

Lower noetics= mathematical objects

Top-down axiomatic approach: this is "the method of geometry and mathematics in general"(Heath 1921, 290): it helps us to prove that **results** are correct (Greenberg 1974, 8) using the **axioms**, which are **never questioned by the user**, and the **logical consequences** we derive from them.

• Bottom-up axiomatic approach: Theoretical Adulthood

When mathematics is applied to the understanding of **complex problems**, it is not anymore based upon **axioms**, which do not require any **reconsideration**. On the contrary, at this level of sophistication, **the consequences of the problem have to be utilized to reconsider the truth of the premises** (Russell 1973, 273-274). In this case, we have not a rational movement which merely goes from an element to its mathematical consideration via a mathematical principle which will not require any reevaluation.

MY COLLABORATION WITH THE TEXT:

FUNCTION of the divided line

Why Plato has chosen *a* schematization to represent intellectual progress?

Why Plato has chosen *that* schematization to represent intellectual progress?

MY COLLABORATION WITH THE TEXT:

FUNCTION OF THE DIVIDED LINE «The oldest and best known discussion of **visual** *discovery* is to be found in Plato's *Meno* (82b-86b)» (Giaquinto 2008, 32. My emphasis).



If a diagonal of one square is a side of another square, this other square has twice the area of the first (Giaquinto 2007, 12)

VISUAL THINKING

VISUAL THINKING

 "In having geometrical concepts for shapes, we have certain general *belief-forming dispositions*. These dispositions can be triggered by experiences of seeing or visual imagining, and when that happens we acquire geometrical beliefs. The beliefs acquired in this way constitute knowledge...provided that the belief-forming dispositions are reliable" (Giaquinto 2007, 12. My emphasis)

In this excerpt Giaquinto explains that a **ViSUal** *discovery* involves the activation of dispositions, that he defines as "belief-forming dispositions" (Giaquinto 2007, 12) that come with possession of certain geometrical concepts (e.g. square, diagonal). What triggers the activation of these dispositions is conscious visual experience. A belief acquired in this way is non-empirical, "because the role of experience is not to provide evidence. At the same time, some visual experience is essential for activating the relevant belief-forming disposition" (Giaquinto 2007, 47. My emphasis)

EPISTEMIC ROLE OF VISUALIZING

VISUAL THINKING

In the case of the *Meno*, one gets the belief almost immediately, that is, "without any subjectively noticeable period between *visualizing* and *getting the belief*. *Immediacy* suggests that to explain why *visualizing* leads to the *belief* we should look to the *visualizer's prior cognitive state*. One hypothesis is that the subject's prior cognitive state included tacitly believing *B*. This kind of view was proposed by Plato. On Plato's view the experience of visualizing triggers retrieval of the tacit belief B" (Giaquinto 2007, 60. My emphasis)

As the whole nature is akin, and the soul has learned everything, nothing prevents a man, after recalling one thing only-a process men call **learning**-discovering everything else for himself, if he is brave and does not tire of the search, for searching and learning are, as a whole, **recollection**. (*Meno*, 81 c-d. My emphasis)

VISUAL THINKING AND THE FUNCTION OF DIAGRAMS

- "the exchange between Socrates and the slave in Plato's *Meno*, approached in the right way, reveals another *role of diagrams*: in this role they enable us to think visually about mathematical subject matter in a way which can be assimilated neither to gathering visual evidence nor to picturing a situation independently described in a proof". (Giaquinto 1993, 81. My emphasis)
- "the use of diagrams in this process is clearly not a superfluous adjunct to a proof (a valid sequence of sentences), since no proof of the theorem was followed or constructed. On the other hand, the use of diagrams was not empirical: the visual experience that resulted from the use of diagrams was not used as a source of observational evidence for this or that proposition. In this case vision was a means of getting information about things that were not before one's eyes." (Giaquinto 1993, 95. My emphasis)

VISUAL THINKING AND THE COLLABORATION BETWEEN WRITER AND READER

- "by following the text supplemented by diagrams, one can **discover for oneself** the geometric theorem as it might have been discovered by the slave if he had complied with Socrates' request to give as answers only what he genuinely believed (83d2) rather than what he guessed Socrates believed; or, if one already knows the theorem, one can see how it **could** be discovered that way by someone not already in the know...So one can approach the text as if the purpose of the exchange between Socrates and the slave is to acquaint or re-acquaint readers with the relevant phenomena of **discovery** through their own experience in following the text." (Giaquinto 1993, 82. My emphasis)
- "by following the exchange and looking at appropriate diagrams you can yourself go through a process of a kind that would lead to discovery if the theorem were new to you. You can imagine that the slave's route to discovery is a Process of just the same type. And you can ask of just that type of process, a process you actually went through in following the text to reach the theorem, how diagrams are used in it". (Giaquinto 1993, 86. My emphasis)

Efficient reasoning is **heterogenous**:

diagrammatic linguistic Representation systems:

HYPERPROOF

The designers of Hyperproof **posed reasoning problems** using *graphical* and *sentential* information.

The givens of the problems are in a diagram window, which is accompanied by sentences in a lower window.

Stenning shows a **diagram window**, which gives an example of the different types of **abstraction** in Hyperproof (Stenning 2002, 56): HETEROGENEOUS REASONING HYPERPROOF

DIAGRAM WINDOW:

ABSTRACTION TRICKS



"*Graphical* **abstraction** in Hyperproof. This diagram contains symbols of varying degrees and **types of abstraction**...The unlabelled cylinder with the question mark badge on the board lacks size, shape and **label** attributes, but still has a **position**; its twin off the chequerboard lacks even a position. The neighbouring medium sized dodecahedron labelled d lacks only a position" (Stenning 2002, 56).

Two blocks appear off to the side of the board. These indicate *not* that there are blocks *off* the board, but rather that there are blocks on the board *in addition* to the ones which are shown there. *Off-board icons* may or may not indicate size and shape but they *abstract over position* (Stenning 2002, 55).

HYPERPROOF AND LOGIC PROBLEMS

- Abstraction tricks: essential for posing reasoning problems
- Reasoning problems: improve the students' general reasoning
 abilities
- Logic: Hyperproof, with its combination of diagrams and sentences,

helps students to see that *logic* is an *abstract*

account of representations

Didactic ladder: hyperproof is a *partially interpreted language*.
 Its predicates have *pre-assigned meanings*.

Partially interpreted formalisms are the

didactic ladder which can be thrown away when the student has ascended it, but without a any aid to climb, the vast majority never grasp anything (Stenning 2002, 62. My emphasis). HYPERPROOF AND LOGIC PROBLEMS

Republic, VI 509d-511:

- Plato gives indication to *trace a line segment* which is the symbol of intellectual development:
 diagram window
- Plato describes the rational faculties which work in each rational step, indicating what are the objects which can be grasped in each reasoning phase: *sentential information*

Republic, VI 509d-511 *is* a logic problem Plato's *schematization* to represent intellectual progress is *part* of a *logic* problem

FUNCTION of the divided line

Why Plato has chosen *a* schematization to represent intellectual progress?

Plato's schematization to represent intellectual progress is a *logic* stimulation of the readers' general reasoning abilities

Plato's schematization to represent intellectual progress is a *logic* stimulation of the readers' *general reasoning abilities*

ABSTRACTION TRICKS:

• DIAGRAM WINDOW IN HYPERPROOF:

two blocks appear *off* the chequerboard: they create the occasion to work on an *abstraction over position*, reflecting on the fact that there are blocks on the board *in addition* to the ones that are shown there.

Republic, VII 534 a:

But as for the ratios between the things these are set over and the division of either the opinable or the intelligible section into two, let's pass them by, Glaucon, *lest they involve us in arguments many times longer than the ones we have already gone through* (My emphasis)

Plato's written words are *not* the *final destination* of knowledge, but rather a stimulation to reach that destination.

• Plato calls for a collaboration between writer and reader

The cognitive stimulation of his readers via *logic* has been devised by Plato.

it is a *partially interpreted language*, as it happens with the *pre-assigned meanings* which characterize the semantics of Hyperproof.

But, as Stenning has pointed to us, this *didactic ladder* can be thrown away when the student has ascended it, but without any aid to this climb, the vast majority of the students never grasp anything (Stenning 2002, 62. My emphasis).

In our case, when the Platonic cognitive stimulation has made us progress intellectually, we can decide to use the rational capabilities that we have sharpened through the Platonic texts, to *criticize*, even radically, his conception of knowledge

IDEAS AND MENTAL MODELS

Plato poses a logic problem to favour our rational growth

What kind of inferences are useful to approach the logic problem posed by Plato?

ADR: when a phenomenon is analyzed with no use of models, the investigation can be developed via abstract direct
 representation, ADR. This kind of representation, as its name suggests, renders *directly* the object of study (Weisberg 2007, 215). An example of ADR is given by Weisberg, who cites
 Mendeleev's Periodic Table, which is the result of the application of ADR on each of the elements which compose it:

Mendeleev examined **elemental properties**, worked out which properties were **essential** and which one could be **abstracted away**, and then constructed a representational system that elucidated important patterns and structure among the elements. This scientific activity constitutes theory construction, but not modelling. **Mendeleev represented chemical phenomena** *directly*, **without the mediation of a model**. (Weisberg 2007, 215)

ADR: investigation of a particular phenomenon in an *immediate* way.

• Scientific models: they are a *medium*, through which complex phenomena are studied *indirectly*.

IDEAS AND MENTAL MODELS: ADR and MODELS THE ANALOGY BETWEEN SCIENTIFIC MODELLING AND PLATO'S IDEAS:

USED TO RENDER THE *EPISTEMIC* **FUNCTION OF THE IDEAS**

The construction of the analogy:

Plato's Ideas have been widely interpreted.

- I am *not* going to take a position with respect to the exegetical realm centred on the interpretation of Plato's Ideas.
- I want to focus my attention on those main characteristics of the Ideas, that can be extended to every Platonic Idea, because they will gradually help us to grasp the association between scientific models and Ideas and this association is crucial to understand the *epistemic function* of the Ideas.

IDEAS AND MENTAL MODELS

IDEAS and MODELS

main characteristics of the Ideas:

Keep as **referent** the excerpt of the *Symposium* quoted below, which presents the characteristics of the Idea of Beauty, that are common to *all* the Platonic Ideas.

This is what it is to go aright, or be led by another, into the mystery of Love: one goes always upwards for the sake of this Beauty, starting out from beautiful *things* and using them like *rising stairs*: from one body to two and from two to all beautiful bodies, then from beautiful bodies to beautiful customs, and from customs to learning beautiful things, and from these lessons he arrives in the end at this lesson, which is learning of this very Beauty, so that in the end he comes to know just *what it is to be beautiful* (*Symposium*, 211 c-d. My emphasis).

In this excerpt Plato explains clearly that you do not abstract the Ideas *directly* from the empirical. Similarly, you cannot reach them *directly* in an intelligible way but you need the empirical stairs to be introduced to the realm of the non-tangible where the Ideas are. The Ideas do *not* function as an ADR of the empirical or the purely intelligible. The Ideas are *abstracta* but they are *not direct* abstractions. The Ideas are the *media* used by Plato to investigate the purely intelligible, where the tangible finds its real significance, what really is.

IDEAS AND MENTAL MODELS

IDEAS and MODELS

The Ideas can be associated with the models

the **models** are the layer constructed to facilitate the investigation of a phenomenon in an *indirect* way. The **Ideas** are the *mediators* elaborated by Plato to ease the readers into the inquiry of the purely intelligible.

 ULTIMATE OBJECTIVES: The investigation of the purely intelligible is for Plato the highest goal that the human mind can have.
 Scientific models are modern tools for the investigation of the empirical.
 Associating Ideas and models I do not want to change their respective

ultimate objectives: the different final destinations of Ideas

and models are not an obstacle for **their association**, which is based on the fact that both Ideas and models are *cognitive artifacts* **devised to ease the** *indirect* **research on complex phenomena**.

IDEAS AND MENTAL MODELS

IDEAS and MODELS

- There is no agreed definition of what is a **mental model** (Rogers and Rutherford 1992, 289).
- I will refer only to the notion of mental models elaborated by Johnson-Laird (Johnson-Laird 1983; 1988):

Mental models, for Johnson-Laird, make us understand the mental processes that occur in *deductive inference* (Johnson-Laird 1983, 23-24)

- In a deduction, the conclusion does not contain more semantic information than the premises.
- In a valid deduction, its conclusion is true in *any* situation in which the premises are true (Johnson-Laird 1988, 219).

IDEAS AND MENTAL MODELS

WHAT IS A MENTAL MODEL? An example of a *deduction* is given by Johnson-Laird, who asks his readers to imagine the following scenario (Johnson-Laird 1983, 23. My emphasis):

Person A asks: Where's the university?

Person B replies: Some of those people are from there.

Person A goes up to the group of people indicated by B and asks them the same question.

A's behaviour depends on a chain of inferences that includes at its centre the following *deduction*:

Some of those people are from the university. Any person from the university is likely to know where the university is.

: Some of those people are likely to know where the university is.

In order to understand how people **make inferences**, for Johnson-Laird, it is necessary to construct a *mental model*. A *mental model* is an *internal tableau* containing **elements** that stand for the **members of sets** (Johnson-Laird 1983, 97-101).

IDEAS AND MENTAL MODELS

WHAT IS A MENTAL MODEL?

CONSTRUCTION OF A MENTAL MODEL: THE FIRST PREMISE

The number of *tokens* corresponding to x's and y's is *arbitrary* (Johnson-Laird 1983, 97).

The number of *entities* depicted is *irrelevant* to any inference that is drawn (Johnson-Laird 1983, 98).

• The *items in parentheses* represent a type that may or may not exist (Johnson-Laird 1983, 97-98).

All of the X are Y: x=y	Some of the X are Y: x=y	None of the X are Y: x	Some of the X are not Y: x
		Х	Х
x=y	x=y		
(y)	(x) (v)	У	(x) =y
(y)		У	У

CONSTRUCTION OF A MENTAL MODEL: THE INFORMATION IN THE SECOND PREMISE IS ADDED TO THE MENTAL MODEL OF THE FIRST PREMISE

- The information in the second premise is added to the mental model of the first premise, taking into account the different ways in which this can be done (Johnson-Laird 1983, 98).
- The principle that motivates the search for alternative ways of adding the information from the second premise is **that in a valid deduction**, the *inference is valid* if and only if there is *no* way of interpreting the premises that is consistent with a denial of the conclusion (Johnson-Laird 1983, 98).
- For some inferences, there is only one possible integrated model (Johnson-Laird 1983, 98):

FIRST PREMISE: SOME OF THE A ARE B	SECOND PREMISE ALL OF THE B ARE C can be <i>integrated only</i> by forming the model:	
a= b	a=b=c	
a= b	a=b=c	ST-S
(a)(b)	(a)(b)= c	
	(c)	

SOME OF THE A ARE C

There is no alternative model of the premises that violates this conclusion (Johnson-Laird 1983, 99).

For other inferences, it is necessary to construct and evaluate two or more models (Johnson-Laird 1983, 99).

CONSTRUCTION OF A MENTAL MODEL: THE CONCLUSION OF THE INFERENCE

In the third and last step of the procedure for making inferences, a conclusion has to be framed to express the relation, if any, between the 'end' terms that holds in all the models of the premises. An 'end' term is one which occurs in only a single premise, unlike the 'middle' term which occurs in both premises. "If there is no such relation between the end terms, the only valid conclusions that can be drawn are trivial ones, such as conjunction or disjunction of the premises, and subjects generally respond that there is no valid conclusion" (Johnson-Laird 1983, 101. My emphasis).

		0	
	FIRST PREMISE:	SECOND PREMISE:	
	SOME OF THE A ARE B	ALL OF THE B ARE C can be <i>integrated only</i> by forming the model:	SOME OF THE
	a= b	a = b = c	A ARE C
	a= b	a = b = c	
	(a)(b)	(a) (b)= c	
Ì		(c)	

THE ANALOGY BETWEEN SCIENTIFIC MODELLING AND PLATO'S IDEAS:

USED TO RENDER THE *EPISTEMIC* **FUNCTION OF** THE IDEAS

«VISION YIELDS MENTAL MODELS»

(Johnson-Laird 1988, 231. My emphasis).

VISUAL THINKING: EPISTEMIC ROLE OF VISUALIZATION

THE VISION INVOLVED IN THE FORMATION OF MENTAL MODELS HAS CLEARLY NOT AN EVIDENCE-PROVIDING ROLE MENTAL MODELS POINT TO AN EPISTEMIC ROLE OF VISION IDEAS AND MENTAL MODELS EMPHASIS ON THE KIND OF *VISION CONSIDERED* EPISTEMICALLY GIVEN BY THE PLATONIC CHOICE OF THE TERM IDEA:

THE WORD *IDEA* COMES FROM THE ROOT *-ID*, THAT IS FOUND IN THE VERBAL FORM *EIDON*, AORIST OF THE VERB *ORAŌ*. THE MEANING OF THIS VERB COMPRISES OF A METAPHYSICAL SHADE: *ORAŌ* MEANS *MENTAL* SIGHT (Liddell et al. 1996, 1245).

PLATO CHOOSES TO REFER TO THE IDEAS USING A WORD WHICH IS *ETYMOLOGICALLY RELATED TO A CONCEPT OF VISION COGNITIVELY TAINTED*. THE VISION OF THE IDEAS IS A VISION THAT MAKES YOU *KNOW*, CONNECTING YOU WITH THE INTELLIGIBLE REALM.

IDEAS AND MENTAL MODELS

A mental number line provides a grasp of the infinity of the natural number structure (Giaquinto 2008, 53. My emphasis).

Obviously we cannot see or visualize more than a finite part of any such line. Thus, **a line with no right end**, **one that continues rightward endlessly**, is the best we can do to render the infinity of the natural number **structure** (Giaquinto 2007, 227. See also Giaquinto 2008, 53).

MENTAL NUMBER LINE

ANTE REM STRUCTURALISM

- The representation above abstracts away from the **nature of the objects**, the **natural numbers**, which instantiate the **natural number Structure**.
- According to Structuralism, numbers, e.g., in the natural number structure, should be treated as positions in structures.
- I want to take into consideration a particular instance of structuralism, Shapiro's ante rem structuralism. The basics of this kind of structuralism are well explained by Sereni:

Arithmetic assertions...are not centred on particular objects...Rather, they are based upon the *positions* of the progression structure. For example, the assertion '3<5' does not state that a particular object, 3, is in the relation 'being minor of' with another particular object, 5. Rather, it states that the *position* of the progression structure that we call '3' (that will be the third or fourth *position* of the structure, according to the fact that we choose to make the structure begin with 1 or 0) comes before, according to the order relation that exists among these *positions*, the *position* of that same structure that we call '5'. *The fact that exist particular objects, numbers, or other abstract objects, or concrete objects, that occupy those positions and that constitute a system that exemplifies the structure in question, is something that lies outside the object of arithmetic and the significance of its assertions. There could exist natural numbers, occupying the positions that we call with their names;...or there could exist nothing that satisfies the relations of the progression structure. Independently from this, the object of arithmetic-that specific structure- does not change,*

and its theorems remain true descriptions of that object. (Sereni 2020, 166-167. My translation. My emphasis)

ANTE REM STRUCTURALISM AND PLATO

- Sereni's words have helped us to understand what ante rem structuralism is: it is a kind of structuralism that ignores the individual properties of the objects, that are irrelevant, and it considers only an object as a position in a structure.
- In ante rem structuralism what is relevant is the universal rather than the empirical; this interest in the universal rather than the empirical, is what connects ante rem structuralism with **Plato's philosophy**, as it is confirmed by Shapiro himself who, in his *Thinking about Mathematics: The Philosophy of Mathematics* (Shapiro 2000, 58-60), considers an excerpt taken from the *Philebus* (*Philebus*, 56 d-e. My emphasis):

Don't we have to agree, first, that the *arithmetic of the many* is one thing, and *the philosophers' arithmetic* is quite another?...*First* there are *those who compute sums of quite unequal units*, such as two armies or two herds of cattle, regardless whether they are tiny or huge. But then there are *the others* who would not follow their example, unless it were *guaranteed that none of those infinitely many units differed in the least from any of the others*.

ANTE REM STRUCTURALISM AND PLATO

- In the passage above Plato emphasizes the difference between *ordinary arithmetic* and *philosopher's arithmetic*. As Shapiro notices, the "*philosopher's arithmetic applies precisely and strictly only to the world of Being*" (Shapiro 2000, 58). Numbers are studied in different ways by philosophers and non-philosophers: "the philosopher's numbers are numbers of pure units" (Shapiro 2000, 59).
- When the philosophers count, as the lines above explain to us, they take into consideration the essence of the units involved in the process of counting. The calculation of the philosophers takes place within the realm of Being, mentioned by Shapiro, where there is **no difference among the units of** the calculation: "Plato's arithmetic is a part of higher philosophy, where one comes to grasp the **metaphysical nature of number itself**" (Shapiro 2000, 60). When the philosophers count, the units involved in this process are the same because the philosopher examines the metaphysical facet of them, their essential aspect and not their contingent appearance. The philosophers count what is essentially homogenous. Differently from the ordinary arithmetician, they know that the heterogeneity of the sensible side of the units counted has to be overcome.

ANTE REM STRUCTURALISM AND PLATO

- Shapiro points to the fact that "ante rem structuralism is a variant of traditional Platonism." (Shapiro 2011, 130. See also Shapiro 2006, 142)
- The "ante rem structuralist takes a Platonic view of structures: they exist and are available for mathematical description as complex objects in their own right, whether or not exemplified by any independent collection of objects" (Wright 2000, 330. My emphasis).
- Shapiro connects ante rem structuralism with Plato's philosophy: for Plato truth is disentangled from the *empirical* realm and can be found in the *purely intelligible*; in the same way, for Shapiro, it is irrelevant the *empirical* existence of objects that exemplify the structures that he is taking into consideration; these objects exist ontologically, as those positions in a structure that can be grasped via an act of intellection. Both for Shapiro and for Plato, the truth is not in the empirical but in the intelligible realm.

ANTE REM STRUCTURALISM AND THEORETICAL ADULTHOOD

- For the structuralist, "mathematics is seen as the investigation...of 'abstract structures', systems of objects fulfilling certain structural relations among themselves and in relation to other systems, without regard to the particular nature of the objects themselves....the 'objects' involved serve only to mark 'positions' in a relational system; and the 'axioms' governing these objects are thought of, not as asserting definite truths, but as defining a type of structure of mathematical interest" (Hellman 2005, 536-537)
- Hellman, as we have seen, has clarified that the axioms, governing the objects that in structuralism are positions in a structure, do not assert definite truths but they define a kind of structure of mathematical interest (Hellman 2005, 537).
- The axiomatic approach connected to structuralism can thus be related to the axiomatic approach that has been called as bottom-up: there are not axioms, that are never questioned, used to logically derive mathematical truths from them; on the contrary, there are axioms whose truth can be reconsidered in light of the results of the mathematical problem examined. This is an axiomatic approach proper of a higher-level of mathematical complexity, pertinent to the investigations of theoretical adults who, as we have seen, analyze the purely intelligible.

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FUNCTION of the divided line

Why Plato has chosen *that* schematization to represent intellectual progress?

Plato has chosen to represent rational development using *that* diagram, a line segment subdivided into sections, to point to us the importance of mathematics in the process of rational growth. Mathematics permeates our entire epistemic development that, in my interpretation of Plato, goes from theoretical childhood to theoretical adulthood. It is pertinent to this advanced stage of rational progress, a kind of mathematics whose interest is the *ontological* existence of its objects, not their correspondent empirical instantiations. I have associated the level of mathematical complexity proper to structuralism with theoretical adulthood

EPISTEMOLOGY IN ANTE REM STRUCTURALISM: THE ACCESS PROBLEM

- **"The access problem"** (MacBride 2008): how can a *physical* being located in a physical universe know the *abstract* realm, that includes *ante rem* **universals** and infinite structures (MacBride 2008)?
- Shapiro's reply (Shapiro 2011, 149. My emphasis) to MacBride's doubts is that

My game, again, is to provide a justification for a philosophical interpretation of mathematics, an interpretation which includes a thesis concerning what mathematics is about-*ante rem* structures. This philosophical interpretation is not a deductive enterprise, where I would have to start with non-mathematical, self-evident premises. *This is a different game from showing a sceptic that mathematics itself is true and known*.

According to Shapiro, the goal of his research is to demonstrate that mathematical knowledge *just* is knowledge of *ante rem* structures. This has not to be proved from accepted *non*-mathematical premises.

 Both Shapiro and Plato do not tell us where their universal evidence comes from. But Plato has chosen to provide us with cognitive stimulations which are entrance points to this epistemic realm. THE COLLABORATION BETWEEN WRITER AND READER

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